

Elliptic-Curve Cryptography and the Internet

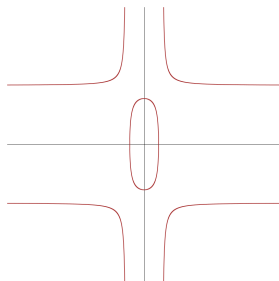
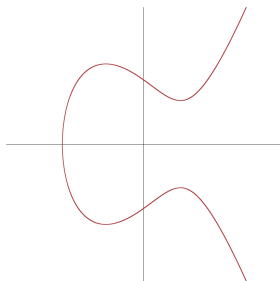
Dominik Joe Pantůček
joe@joe.cz

Trustica s.r.o.

7.12.2015

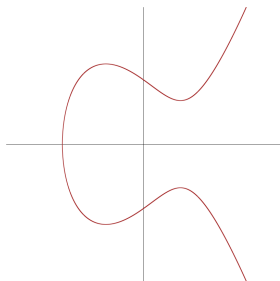
What is EC?

- Elliptic curve
- Looks nice



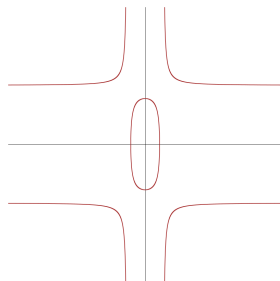
- 1D abelian variety
- Forms abelian group over 2D finite field

What is EC?



$$y^2 = x^3 - 2x + 2$$

EC in Weierstrass form.



$$10x^2 + y^2 = 1 + 6x^2y^2$$

Twisted Edwards curve.

What is ECC?

- Elliptic-curve cryptography
- Public-key cryptography system
- Form abelian group over 2D finite field
 - Point addition: $C = A + B$
 - Point duplication: $B = A + A$
 - Point multiplication: $B = n \cdot A$
- Discrete logarithm problem on the EC group:
 - Given points A and B , find n such that $B = n \cdot A$.
- Good luck ...

Why should we care?

- Security parameter:
 - The bigger the better ...
 - 80bit security is roughly 1024bit RSA
 - The number of required brute-force attempts is approximately 2^n
 - 2^{80} for 1024bit RSA
- Size:
 - The smaller the better ...
 - 1024bit RSA keys and values are 1024 bits long, i.e. 128 bytes
 - Speed and/or latency impact during key exchange

Why should we care?

Elliptic-curve cryptography:

- security parameter is $\frac{n}{2}$
- n is typically 256 – 256bits per coordinate (2D)
- only one coordinate is needed
- +1 bit of the other one for checksum calculation
- $\frac{257}{8} = 33$ bytes for 128-bit security

What ECC is available?

- Curves:
 - NIST – National Institute of Standards and Technology
 - SECG – Standards for Efficient Cryptography Group
 - Independent (Bernstein et al.)
- Authentication: ECDSA
- Encryption: ECDH/ECDHE
 - Parameters: p, a, b, G, n
 - Key pairs: d_A, Q_A and d_B, Q_B with $Q_A = d_A G$ and $Q_B = d_B G$
 - Commutativity: $d_A d_B G = d_B d_A G$
 - Shared secret: $d_A Q_B = d_B Q_A$

Standard curves and algorithms

```
$ openssl ecparam -list_curves
```

```
secp112r1 : SECG/WTLS curve over a 112 bit prime field
secp112r2 : SECG curve over a 112 bit prime field
secp128r1 : SECG curve over a 128 bit prime field
secp128r2 : SECG curve over a 128 bit prime field
secp160k1 : SECG curve over a 160 bit prime field
secp160r1 : SECG curve over a 160 bit prime field
secp160r2 : SECG/WTLS curve over a 160 bit prime field
secp192k1 : SECG curve over a 192 bit prime field
secp224k1 : SECG curve over a 224 bit prime field
secp224r1 : NIST/SECG curve over a 224 bit prime field
secp256k1 : SECG curve over a 256 bit prime field
secp384r1 : NIST/SECG curve over a 384 bit prime field
secp521r1 : NIST/SECG curve over a 521 bit prime field
...
```


- Curve25519

$$y^2 = x^3 + 486662x^2 + x \pmod{2^{255} - 19}$$

- Ed25519

$$-x^2 + y^2 = 1 - \frac{121665}{121666}x^2y^2 \pmod{2^{255} - 19}$$

- X25519 (DH)
- EdDSA

Library support

- OpenSSL
- Microsoft Cryptography API: Next Generation
- LibreSSL
- GNUTLS
- NaCl

- Since 0.9.8 (5 Jul 2005)
- ECDSA – ECDSA_* *-ECDSA-*
- ECDH – ECDH_* ECDH-*
- Example (configuration string): ECDH-ECDSA-AES256-GCM-SHA384

Microsoft Cryptography API: Next Generation

- Windows Vista and newer, Windows Server 2008 and newer
- ECDSA – `BCRYPT_ECDSA_P256_ALGORITHM`
- ECDH – `BCRYPT_ECDH_P256_ALGORITHM`
- Example: `TLS_ECDHE_ECDSA_WITH_AES_256_CBC_SHA_P384`

- Forked from OpenSSL by OpenBSD folks in 2014
- API is the same
- New (preferred) libtls API uses same configuration strings
- Example (configuration string): ECDH-ECDSA-AES256-GCM-SHA384

- GNU implementation
- Since 3.0.0. (29 Jul 2011)
- Configuration strings – called “priorities”
- Example: ECDH-ECDSA-AES256-GCM-SHA384

- [Salt]
- Daniel J. Bernstein et al.
- Ed25519 in the next release
- May support NIST-P256/SHA-512-based ECDSA

Application support

- Servers
- Clients

HTTP server support

- Apache
- IIS
- Nginx
- LigHTTPd

HTTP server support

- Supported only in TLS 1.0 and higher.
- Disabling SSL 2.0 and 3.0 is strongly recommended.

- mod_ssl links against OpenSSL

```
SSLCipherSuite ECDHE-ECDSA-AES128-GCM-SHA256  
SSLProtocol All -SSLv2 -SSLv3
```

- Supported since Windows Server 2008
- Supports ECDHE_RSA
- ECDHE_ECDSA only with ECC certificates
- Cipher preferences (order) can be configured in:
 - SSL Cipher Suite Order policy

- Linked against OpenSSL

```
ssl_ciphers "ECDHE-ECDSA-AES128-GCM-SHA256"  
ssl_protocols TLSv1 TLSv1.1 TLSv1.2;
```

- Once again ... linked against OpenSSL

```
ssl.cipher-list = "ECDHE-ECDSA-AES128-GCM-SHA256"  
ssl.use-sslv2 = "disable"  
ssl.use-sslv3 = "disable"
```

HTTP client support

- Firefox
- Chrome
- Microsoft Internet Explorer

- Uses NSPR and NSS
- Fully supported at least since 2009
- All builds from last 5 years ...
- No manual configuration needed

- Fully supported with the same backend as Firefox
- All builds from last 5 years ...
- No manual configuration needed

- Fully supported since version 9
- No manual configuration needed

- SSH/OpenSSH
- OpenPGP/GnuPG

- ECDSA and ECDH supported since version 5.7 [23 Jan 2011]
- Ed25519 supported since 7 Dec 2013 snapshots
- Official support since version 6.5 [2 Feb 2014]
- It is preferred type of both server and client keys now

```
ssh-keygen -t ed25519 -C My-Ed25519-key -f generated-key
```

- Supports standardized ECC
- NIST P-256, P-384, and P-521 curves
- Brainpool P-256, P-384, and P-512 curves
- Both for signing and encryption

Experimental Ed25519/EdDSA support

- draft-koch-eddsa-for-openpgp-03 [Aug 28, 2015]
 - 22 is probable id
 - expires [Feb 29, 2016]
- draft-josefsson-eddsa-ed25519-03 [May 12, 2015]
 - expired [Nov 13, 2015]
- draft-irtf-cfrg-eddsa-00 [Oct 7, 2015]
 - expires [Apr 9, 2016]

OpenPGP/GnuPG

```
$ gpg --expert --full-gen-key
```

```
...  
Please select what kind of key you want:
```

```
...  
  (10) ECC (sign only)
```

```
...  
Your selection? 10
```

```
Please select which elliptic curve you want:  
  (1) Curve 25519
```

```
...  
Your selection? 1
```

```
gpg: WARNING: Curve25519 is not yet part of the OpenPGP  
Use this curve anyway? (y/N) y
```

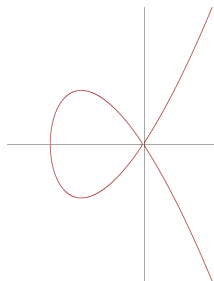
```
...
```

Conclusion

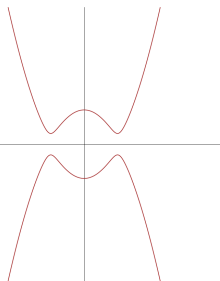
By using ECC wisely you...

- get bigger security parameter,
- get smaller and faster handshakes, and
- make sure the future will be pretty twisted.

Questions



Feel free to ask some ;-)



References

- Elaine Barker, Lily Chen, Allen Roginsky and Miles Smid: NIST Special Publication 800-56A Revision 2 – Recommendation for Pair-Wise Key Establishment Schemes Using Discrete Logarithm Cryptography, May 2013
- NIST: FIPS PUB 186-4 – Digital Signature Standard (DSS), July 2013
- IETF work in progress drafts:
 - W. Koch: EdDSA for OpenPGP (draft-koch-eddsa-for-openpgp-03), August 28, 2015
 - S. Josefsson and N. Moeller: EdDSA and Ed25519 (draft-josefsson-eddsa-ed25519-03), May 12, 2015
 - S. Josefsson and I. Liusvaara: Edwards-curve Digital Signature Algorithm (EdDSA) (draft-irtf-cfrg-eddsa-00), October 7, 2015
- Me et al.: all elliptic curve images presented, December 2015

Thank you.